

## **THE WELFARE EFFECTS OF PRIVATIZATION AND SUBSIDIZATION IN A MIXED MARKET FOR COMPLEMENTARY GOODS**

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**Abstract:** This research examines the welfare effects of production subsidies in a mixed market with complementary goods, where a state-owned public firm and a private firm produce complementary products. The study considers four regimes: unsubsidized mixed duopoly, subsidized mixed duopoly, unsubsidized private duopoly, and subsidized private duopoly. The paper presents two main results. First, if production subsidies are used only before privatization, then there is a decrease in social welfare. Second, if production subsidies are used before and after privatization, then social welfare is not changed by privatization. The analysis shows that these results are consistent with previous research on production subsidies in a Cournot mixed market with homogeneous goods. The study also provides equilibrium outputs and profits for each of the four regimes, highlighting the differences in consumer surplus, social welfare, and profit for each scenario. This research contributes to the literature on subsidy policies and privatization in mixed markets, particularly for complementary goods.

**Keywords:** production subsidies, mixed market, complementary goods, privatization, social welfare, Cournot model.

### **Introduction**

As is very well known, the trend of privatization of public firms has been increasing worldwide since the nineteen-eighties. Therefore, the theoretical research on privatization is often done in the context of mixed oligopoly models where private firms compete with state-owned public firms (for example, see Anderson, de Palma, & Thisse, 1997; Bárcena-Ruiz & Garzón, 2005; Bosi, Girmens, & Guillard, 2005; Chang, 2005; Chao & Yu, 2006; Dadpay, 2014; Fjell & Pal, 1996; George & La Manna, 1996; Gronberg & Hwang, 1992; Han & Ogawa, 2008; Kato & Tomaru, 2007; Kim, Lee, & Matsumura, 2019; Mujumdar & Pal, 1998; Myles, 2002; Ohnishi, 2012, 2021; Pal & White, 1998; Poyago-Theotoky, 2001; Sasaki & Wen, 2003; Wang, Wang, & Zhao, 2009; White, 1996). For instance, White (1996) presents three effects of production subsidies in a quantity-setting mixed oligopoly market. First, if production subsidies are utilized before and after privatization of a state-owned public firm, then privatization does not change social welfare. Second, if production subsidies are used before but not after privatization, then there is a reduction in social welfare. Third, the production subsidy contributes to overall efficiency in a mixed market because of cost distribution effects. Poyago-Theotoky (2001) and Myles (2002) show that the optimal production subsidy is identical irrespective of whether (i) a state-owned public firm moves simultaneously with  $n$  private firms, (ii) it acts as a Stackelberg leader, or (iii) all firms behave as profit-maximizers. In addition, Ohnishi (2012) studies the

welfare effects of production subsidies in a price-setting mixed duopoly market by extending the analysis by White (1996), and finds that the introduction of production subsidies into the analyses of Cournot and Bertrand mixed markets can improve social welfare. However, most studies examine privatization in mixed markets where public and private firms produce homogeneous or substitute products.

In this paper, we consider a Cournot mixed market in which a state-owned public firm and a private firm produce complementary products, and reassess the welfare effects of production subsidies regarding privatization. We examine four regimes: mixed and private duopoly, each with and without subsidies. In the regimes without subsidies, a one-shot Cournot-Nash game is constructed. In the regimes with subsidies, the following two-stage game is examined: At the first stage the government chooses the subsidy to maximize social welfare, and at the second stage each firm observes the subsidy and noncooperatively chooses its quantity level. We solve and compare the four regimes. The main purpose is to examine the welfare effects of production subsidies in a mixed market with complementary goods regarding privatization.

## Basic Setting

We consider an industry in which there are one state-owned welfare-maximizing public firm (firm 0) and one profit-maximizing private firm (firm 1). In the remainder of this paper, subscripts 0 and 1 denote firm 0 and the firm 1, respectively. In addition, when  $i$  and  $j$  are used to refer to firms in an expression, they should be understood to refer to 0 and 1 with  $i \neq j$ . There is no possibility of entry or exit. Both firms produce complementary goods. There is a continuum of consumers of the same type, and the representative consumer maximizes consumer surplus:  $CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1$ , where  $q_i$  is the amount of good  $i$  and  $p_i$  is its price. The function  $U(q_0, q_1)$  is quadratic and symmetric in  $q_0$  and  $q_1$ :  $U(q_0, q_1) = a(q_0 + q_1) - (q_0^2 - 2bq_0q_1 + q_1^2)/2$ , where  $a \in (0, \infty)$  is a constant and  $b \in (0, 1)$  is a measure of the degree of complementarity among products.

The inverse demand function is given by

$$p_i = a - bq_i - bq_j, \quad (1)$$

For the sake of simplicity, we assume  $b = 0.5$ . Both firms have the same production function represented by a quadratic cost function:  $C(q_i) = c + q_i^2/2$ . We assume  $c = 0$ , since entry decisions are not considered.

Therefore, each firm's profit is given by

$$\pi_i = p_i q_i - c - \frac{1}{2} q_i^2 + s q_i, \quad (2)$$

where  $s \in (0, \infty)$  denotes the subsidy for each unit of output.

Social welfare, defined as the sum of consumer surplus and producer surplus, is given by

$$W = CS + \pi_0 + \pi_1 = a(q_0 + q_1) - \frac{1}{2}(q_0^2 - 2bq_0q_1 + q_1^2) - p_0 q_0 - p_1 q_1 + \pi_0 + \pi_1. \quad (3)$$

We examine four regimes: mixed and private duopoly, each with and without subsidies. In the regimes without subsidies, a one-shot Cournot-Nash game is considered and solved. In the regimes with subsidies, a two-stage Cournot game is considered: At the first stage the government chooses the subsidy level to maximize social welfare, and at the second stage each firm observes the subsidy and non-cooperatively determines its output level.

## Results

In this section, we discuss the following four games: unsubsidized mixed duopoly, subsidized mixed duopoly, unsubsidized private duopoly, and subsidized private duopoly.

### Unsubsidized Mixed Duopoly

We present the Cournot equilibrium values of outputs, profits, consumer surplus and social welfare when there is no subsidy:

$$q_1^M(0) = \frac{10}{23}a, \quad q_0^M(0) = \frac{14}{23}a, \quad (4)$$

$$Q^M(0) = \frac{24}{23}a, \quad (5)$$

$$\Pi_1^M(0) = \frac{529}{150}a^2, \quad \Pi_0^M(0) = \frac{529}{98}a^2, \quad (6)$$

$$CS^M(0) = \frac{78}{529}a^2, \quad (7)$$

$$W^M(0) = \frac{529}{326}a^2. \quad (8)$$

Notice that firm 0 makes a strictly positive profit.

### Subsidized Mixed Duopoly

In this subsection, we examine the mixed duopoly game when the government considers setting a production subsidy. The timing of the game is as follows. At stage one, the government chooses the subsidy level to maximize social welfare. At stage two, each firm simultaneously and independently chooses its output level conditional on the subsidy. We discuss the subgame perfect Nash equilibrium by backward induction. Maximizing (2) and (3) simultaneously, we obtain the second-stage Cournot equilibrium outputs for a given subsidy:

$$q_1^M(s) = \frac{10a + 8s}{23}, \quad q_0^M(s) = \frac{14a + 2s}{23}, \quad (9)$$

We now consider the first stage of the game. At the first stage, the government takes into account how firms will react to the subsidy and sets the subsidy level to maximize (3). We can obtain the welfare-maximizing subsidy as follows:

$$s^M = \frac{2a}{3}. \quad (10)$$

Notice that  $s^M$  is strictly positive. From (9) and (10), we obtain the following subgame perfect Nash equilibrium values:

$$q_1^{s^M}(s^M) = q_0^{s^M}(s^M) = \frac{2}{3}a, \quad (11)$$

$$Q^{s^M}(s^M) = \frac{4}{3}a, \quad (12)$$

$$\Pi_1^{s^M}(s^M) = \Pi_0^{s^M}(s^M) = \frac{1}{3}a^2, \quad (13)$$

$$CS^{s^M}(s^M) = \frac{1}{9}a^2, \quad (14)$$

$$W^{s^M}(s^M) = \frac{1}{9}a^2. \quad (15)$$

W 3

From the preceding results, we can have the following proposition.

**Proposition 1:** If optimal production subsidies are used in a mixed duopoly, then  $q_1^M(s^M) = q_0^M(s^M)$ ,  $Q^M(0) < Q^M(s^M)$ , and  $W^M(0) < W^M(s^M)$ .

### Unsubsidized Private Duopoly

In the subsection, we present the one-shot Cournot equilibrium outcomes of the unsubsidized private duopoly game. Each duopolist maximizes its own profit. Therefore, we can obtain the following equilibrium values:

$$q_0^P(0) = q_1^P(0) = \frac{2}{5}a, \quad (16) \quad Q^P(0) = \frac{4}{5}a, \quad (17)$$

$$\pi^P(0) = \pi_0^P(0) = \frac{2}{25}a^2, \quad (18)$$

$$CS^P(0) = \frac{2}{25}a^2, \quad (19)$$

$$W^P(0) = \frac{2}{25}a^2. \quad (20)$$

We compare the subsidized mixed duopoly outcomes with those of the unsubsidized private duopoly. We see that social welfare is higher in the subsidized mixed market than in the unsubsidized private market. We now present the following proposition.

**Proposition 2:** If optimal production subsidies are used only before privatization of firm 0 in a mixed market, then  $W^M(s^M) > W^P(0)$ .

### Subsidized Private Duopoly

In this subsection, we consider the following two stage game. At stage one, the government chooses the output subsidy level to maximize social welfare. At stage two, each profit-maximizing firm simultaneously and independently chooses its output level conditional on the subsidy. The game is solved by backward induction to obtain a subgame perfect Nash equilibrium. Starting from the second stage, we obtain the Cournot equilibrium outputs for a given subsidy:

$$q_1^P(s) = q_0^P(s) = \frac{14(a+s)}{35}. \quad (21)$$

We consider the first stage of the game. In the first stage, the government takes into account how firms will react to the subsidy and determines the subsidy level to maximize (3). It happens that the optimal subsidy, outputs, profits, consumer surplus and social welfare in this game are the same as those in the subsidized mixed duopoly:  $s^P = s^M$ ,

$$q_1^P(s^P) = q_0^P(s^P) = q_1^M(s^M) = q_0^M(s^M), \quad Q^P(s^P) = Q^M(s^M), \quad \pi_1^P(s^P) = \pi_0^P(s^P) = \pi_1^M(s^M) = \pi_0^M(s^M),$$

$$CS^P(s^P) = CS^M(s^M), \quad \text{and} \quad W^P(s^P) = W^M(s^M).$$

Therefore, the effects of the subsidy on the private duopoly results are stated by the following proposition.

**Proposition 3:** If optimal production subsidies are used in a Cournot private market, then  $Q^P(0) < Q^P(s^P)$ , and  $W^P(0) < W^P(s^P)$ .

Finally, we compare the two subsidized games. The optimal subsidy, outputs, profits, consumer surplus and social welfare in the subsidized mixed market are identical with those in the subsidized private market. This comparative result is summarized in the following proposition.

**Proposition 4:** If optimal production subsidies are used before and after privatization of firm 0 in a quantity-setting mixed market, then  $s^M = s^P$ , and  $W^M(s^M) = W^P(s^P)$ .

## Conclusion

We have examined the welfare effects of production subsidies in a mixed market where a state-owned public firm and a private firm produce complementary goods, and have considered the following four regimes: unsubsidized mixed duopoly, subsidized mixed duopoly, unsubsidized private duopoly, and subsidized private duopoly. We have found that our results are the same as those obtained by the existing Cournot mixed market model with homogeneous goods.

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