CASE STUDIES IN ESTIMATING WIND POWER POTENTIAL: USING MAXIMUM LIKELIHOOD AND MODIFIED MAXIMUM LIKELIHOOD METHODS

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Abstract: Wind power is an important renewable energy source, and estimating its potential is necessary for efficient utilization. In this paper, we discuss various methods for estimating wind power potential, with a focus on the use of the Weibull distribution. We present five approaches for estimating the parameters of the Weibull distribution, including the Maximum likelihood method, the Modified maximum likelihood method, Error of approximation, Method of Moment, and the Energy pattern factor method. A sample wind speed dataset is used to compare the accuracy of each method. We also highlight the importance of wind power forecasting in grid-connected wind generating plants and discuss various modern methodologies for weather forecasting. Lastly, we present a case study for the Energy pattern factor method, demonstrating its applicability in wind energy production.

Introduction:

Renewable energy sources have become essential in today's world, and wind power is one of the most promising options. For efficient utilization of wind power, it is necessary to estimate its potential accurately. Various methodologies, including statistical models, have been proposed to estimate wind power potential. In this paper, we focus on the use of the Weibull distribution for wind power potential estimation. We describe the five approaches for estimating the parameters of the Weibull distribution, including the Maximum likelihood method, the Modified maximum likelihood method, Error of approximation, Method of Moment, and the Energy pattern factor method. A comparative analysis of these methods is presented using a sample wind speed dataset. We also emphasize the significance of wind power forecasting in grid-connected wind generating plants and discuss various modern methodologies for weather forecasting. Lastly, we present a case study for the Energy pattern factor method, demonstrating its direct applicability in wind energy production. Overall, this paper provides a comprehensive understanding of wind power potential estimation using the Weibull distribution and its associated methodologies, which can aid in the efficient utilization of wind energy. Wind forecast models are of two categories:

● *Numerical based model.* ● *Statistical model.*

In physical measurements, changing dynamics such as temp, wind speed, relative humidity, and pressure, as well as the requirement for bigger resources, numerical approaches use meteorological characteristics, geographical features, altitude, durability, and restrictions. Forecasting services are also available from other qualified firms. Statistical models (Chandel et al., 2014) use meteorological data to anticipate wind speed of wind in future and wind's output power, requiring a single step only to transform input variables to output power. Autoregressive (AR), Moving average (MA), integrated motion measurement model (ARIMA), Box-Jenkins method, Kalman-filtering, and Artificial Neural Network (ANN) etc (A.M & Leahy, P.G., 2012). The Potential estimate of wind power usually depends on years-term meteorological measurements in the area of interest. In a comprehensive evaluation, In paper (Jung & Broadwater, 2014, 762-777) a precise technique was presented for assessing a region's wind resource and producing a wind map for the region. After finding acceptable windy places, (Aggarwal et al., 2014) conducted a preliminary assessment of Himachal Pradesh's wind potential, which was again acted on by the detailed assessment programme.

Chang (Chang, 2011) studied six numerical approaches for calculating wind energy patterns and found that the WEPF technique is superior in estimating Weibull parameters. The importance of utilizing wind resources in Himachal Pradesh for energy generation and other purposes is stressed upon in (Aggarwal et al., 2014), but no big initiatives have yet been implemented. (WEPF) approach was utilized for the potential assessment of Himachal Pradesh. Various methods for the calculation of Weibull Parameters have been given in (Lars Lundberg et al., 2003) like the Moment method, Maximum likelihood method, Modified maximum likelihood, Energy Estimation etc., for wind energy application in Iran's cities.

From all the methods given the energy pattern factor method has less calculations compared to other methods and there are no numerical iterations in EPF method, we simply need to calculate the annually or monthly mean wind speed and then from that mean wind speeds EPF are then calculated annually or monthly (Dogara & Aboh, 2016).

The method's applicability is determined by the sample size, distribution and wellness of fit tests (Akdag and Ali, 2009) This frequency distribution technique can easily anticipate the wind energy conversion system's output energy. It has been proven to function with a wide range of wind data.

2. Weibull Distribution

In reliability engineering, the Weibull distribution (Al-Hinai et al., 2021) is one of the most commonly utilized lifespan distributions. It is a flexible distribution that, depending on the values of the parameters, may take on the properties of other types of distributions. In recent years, this technique has garnered a lot of interest for wind energy applications, not only because of its enhanced flexibility and simplicity, but also because of its ability to create an excellent fit to experimental data. The mathematical representation of the 2-parameter Weibull distribution function is (Dodla, 2018).

$$f(v) = \left[\left(\frac{k}{c}\right)\right]\left[\left(\frac{v}{c}\right)\right]^{k-1} e^{\left[-\left(\left(\frac{v}{c}\right)\right)\right]^k} \tag{1}$$

and cumulative density is expressed as:

$$F(v) = e^{\left[-\left(\left(\frac{v}{c}\right)\right)\right]^k} \tag{2}$$

where 'v.' is the speed at which wind blows, shape parameter is defined by the symbol 'k' and scale parameter is defined by the symbol 'c'.

The two Weibull parameter and the average wind speed and the Parameters of weibull distribution are related with the help of given expression:

$$\bar{v} = c\Gamma(1 + (\frac{1}{k}))$$
(3)

where '$\bar{v}$' is the average of wind velocity.

2.1Wind speed data: Wind speed data is generally available in two formats, one is time series data in which the data is available in data points and each data point represent the instantaneous value or average value of wind speed (Langreder, 2010) over some defined time period and the second is frequency distribution format in which the data is divided into different bins(ranges).The methods described below can be used to estimate the 'k' and 'c' when wind speed is available in either format.

3. Weibull Parameters' Determination

Five methods of estimation of Weibull distribution have been presented: Maximum likelihood, Modified Maximum likelihood, Error of Approximation, Moment method and Wind Energy Pattern Factor Method.

3.1Maximum likelihood method: It is an approach by which we can calculate the values of a model's parameters as the maximum likelihood estimate. The parameter values are set to increase the likelihood that the model's indicated process created the data that were actually observed. Maximum likelihood estimation (Chang, 2011) is a method for calculating the parameters whose values produce the best-fitting curve for the data.

Table 1. Data of wind velocity in time series (Dodla, 2018) format

Time (Hr.)	Wind velocity(m/s)	Time(Hr.)	Wind velocity(m/s)	Time(Hr.)	Wind velocity(m/s)	Time(Hr.)	Wind velocity(m/s)
1 hr	3.7	7 hr	4.65	13 hr	5.77	19 hr	4.34
2 hr	3.23	8 hr	2.77	14 hr	8.3	20 hr	3.79
3 hr	4.21	9 hr	5.21	15 hr	9.2	21 hr	4.2
4 hr	3.35	10 hr	6.71	16 hr	9.3	22 hr	2.8
5 hr	2.82	11 hr	6.83	17 hr	6.56	23 hr	3.7
6 hr	3.14	12 hr	6.81	18 hr	4.25	24 hr	3.3

The parameters 'k' and 'c' are estimated using equations (4) and (5)

$$k = \left[\left(\frac{\sum_{i=1}^{n} v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k}\right) - \left(\frac{\sum_{i=1}^{n} \ln(v_i)}{n}\right)\right]^{-1}$$
(4)

$$c = \left(\frac{1}{n}\left(\sum_{i=1}^{n} v_i^k\right)\right)^{\frac{1}{k}}$$
(5)

where 'v_i' is the wind speed in the time step 'i' and 'n' is the no. of measurements.

- Maximum likelihood method requires a large number of numerical iterations (The initial guess which is most suitable isk=2).
- For tiny samples, maximum likelihood estimates might be severely skewed.
- For small samples optimal properties are not applicable.

3.2   Modified maximum likelihood method: The modified (Chang, 2011) approach is identical to the regular maximum likelihood (ML) method, with the added benefit of improved convergence and robustness. This makes it particularly beneficial for identification in situations when other methods have failed due to the existence of outliers.

Table 2. Data of wind velocity (Dodla, 2018) in frequency distribution format

Wind Velocity(m/s)	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
In Frequency form (%)	4	8	10	16	21	18	9	1	0	0

The Parameters of Weibull can be estimated using the equations (6) and (7) shown below:

$$k = \left[\left(\frac{\sum_{i=1}^{n} v_i^k f(v_i) \, ln(v_i)}{\sum_{i=1}^{n} f(v_i) \, v_i^k} \right) - \left(\frac{\sum_{i=1}^{n} ln(v_i) f(v_i)}{f(v_i \geq 0)} \right) \right]^{-1} \tag{6}$$

$$c = \left(\frac{1}{f(v_i \geq \sim 0)} \left(\sum_{i=1}^{n} f(v_i) \, v_i^k \right) \right)^{\frac{1}{3}} \tag{7}$$

where the Weibull distribution function is represented by 'f(v)' , the function of cumulative density is represented by 'F(v)', the instantaneous wind speed is 'v$_i$' ● If a frequency distribution data sample is available, we may utilize this approach. ● Added benefit of improved Convergence and Robustness.

3.3   Equivalent energy Method: With Approximation (Chang, 2011) error 'k' and 'c' parameters are estimated.

$$\sum_{i=1}^{n} \left[W_{vi} - e^{-\left[\frac{(v_i - 1)[\Gamma(1 + \frac{3}{k})]^{\frac{1}{3}}}{[(v_m^3)^{\frac{1}{3}}]^k} \right]} + e^{-\left[\frac{(v_i)[\Gamma(1 + \frac{3}{k})]^{\frac{1}{3}}}{[(v_m^3)^{\frac{1}{3}}]^k} \right]} \right] = \sum_{i=1}^{n} \left[\varepsilon_{vi}^2 \right] \tag{8}$$

"Energy density is a parameter that helps determine the Weibull distribution parameters for wind energy applications," says the first hypothesis. The corresponding deterministic factor portion must conform to the energy content equivalence criteria found in the Weibull distribution, and the scale factor 'c' can be expressed as the mean cube expression.

$$c = \left[\frac{v_m^3}{\Gamma(1 + \frac{3}{k})} \right]^{\frac{1}{3}} \tag{9}$$

3.4   Moment Method: Pafnuty Chebyshev established the method of moments in the proving of the central limit theorem in 1887. The method of moments (Chang, 2011) is a method of parameter estimation. It begins by expressing the predicted values of powers of the random variable in question as functions of the relevant parameters. The sample moments are then put equal to those expressions.

$$\bar{v} = c\Gamma(1 + (\frac{1}{k})) \sigma = c\Gamma[(1 + (\frac{2}{k}) - \Gamma^2(1 + \frac{1}{k})]^{\frac{1}{2}}) \tag{10}$$

where

$$\bar{v} = (\frac{1}{n} \sum_{i=1}^{n} v_i), \sigma = \left[\frac{1}{n-1} \sum_{i=1}^{n} (v_i - \bar{v}^2) \right]^{\frac{1}{2}} \Gamma(X) = \left[\int_0^\infty t^{(x-1)} exp(-t) \, dt \right]^{\frac{1}{2}}$$

●   One of the oldest estimating methods is the moment method.

●   The first and second moments of the distribution near zero are used to estimate Weibull parameters.

●   The advantage is that it is simple; the downside is that it lacks the required optimality properties as that of other methods.

●   This method generally estimates starting values for maximum likelihood and least squares estimations so that the result will be more precise.

The wind power density in terms of Weibull distribution is given by:

$$WPD = \frac{1}{2}\, \rho\, c^3\, \Gamma\left[1 + \frac{3}{k}\right] \quad (11)$$

where 'WPD' is the Wind Power Density, shape of distribution is defined by 'k' parameter, scale is defined by the 'c' parameter and 'ρ' is air density.

3.5 Energy Pattern factor Method: For wind resource assessments in wind energy applications, a variety of statistical approaches are used. To assess the wind power potential of various sites using averaged wind speed we can use a new technique or method known as Wind energy pattern factor [WEPF] technique. The WEPF (Akdag & Dinler, 2009) approach has been shown to be adequate for reliably predicting Weibull parameters (Majid et al., 2015). The method's simplicity stems from the fact that it does not require binning or the solution of a linear least square problem, nor does it use an iterative strategy.

● If we have available wind velocity data in average form we can calculate WEPF.

● Based on the mean yearly or monthly wind speed, this figure can be used to compute the available energy in the wind.

● Many years of data from nearby stations can be combined with short-term on-site measurements, which is valuable in locations where wind data is scarce.

$$E_{pf} = \left[\left(\frac{\bar{v}^3}{\bar{v}^3}\right)\right], \; k = \left[1 + \left(\frac{3.69}{E_{pf}^2}\right)\right] \bar{v} = c\, \Gamma\left[1 + \left(\frac{1}{k}\right)\right] \quad (12)$$

Mean wind speed is a good indicator of a location's wind resource; the Power Density provides a more accurate picture of a location's wind energy potential.

The wind's power at speed "v" with a blade swept area "A" grows as the cube of its velocity and is given by:

$$P = \left(\frac{1}{2}\, \rho\, A\, v^3\right) \quad (13)$$

where "P" stands for the "WPD" (W/m2) and 'ρ' is the air density (kg/m3), which is influenced by height, air pressure, and temperature. The rotor area is "A", and the wind speed is "v" (m/s). At sea level, the density of air is calculated to be 1.225 kg/m3. Because of the considerable variation in speed of the wind, to increase accuracy, the Energy Pattern Factor (EPF) as a correction factor may be applied.

$$WEPF = \frac{1}{n\bar{v}^3} \sum_{i=1}^{n} \bar{v_i}^3 \quad (14)$$ where the instantaneous speed of wind in meters per second is 'v_i', the mean wind speed in meters per second is 'v' and 'n' is the

measurement of the data point. WEPF can have a value ranging from zero to four. The following is another definition of the WEPF:

$$WEPF = \frac{Average\ of\ the\ cubes\ of\ wind\ Speed}{Cube\ of\ the\ average\ of\ wind\ speed} = \left[\frac{\bar{v}^3}{\bar{v}^3}\right] \quad (15)$$

Therefore, For a certain location in terms of WEPF the average Wind power density (WPD) is expressed as:

$$WPD = \frac{1}{2}\, (W.E.P.F\, \rho\, \bar{v}^3) \quad (16)$$

4. Results and Discussion

Let us consider one case study. The table below shows the wind speed for 12 months. If we calculate the Weibull parameters from the following case study we can see a very slight variation in the parameters, but this slight variation causes a lot of difference.

Table 3. Mean wind speeds (Fasel et al., 2021) for different Months

Month	Mean wind velocity (m/s)	Month	Mean wind velocity (m/s)	Month	Mean wind velocity (m/s)
January	5.62	May	4.52	September	9.27
February	6.36	June	7.89	October	8.84
Mach	5.85	July	9.43	November	6.68
April	4.48	August	10.18	December	4.68

Figure 1. Wind speed variation (Akdag & Dinler, 2009) for different months

As per the data referred to above, the mean wind speed is highest in the month of August and is found to be 10.1719 m/s and the lowest wind speed is in the month of April which is found to be 4.4753 m/s. This variation in wind speed is due to the temperature difference, As the wind speed is varying (Fasel et al., 2021) so the wind Power density is also variable as it depends on the cube of the mean wind speed. The values of k and c for different methods are calculated using the equations shown in the respective method, it is seen that the values of k and c for different methods are nearly the same , but the small change can cause a lot of difference.

Table 4. Calculation (Indhumathy and Seshaiah) of 'k' and 'c' for different methods

Weibull Parameters	Maximum Likelihood Method	Modified Maximum Likelihood Method	Moment Method	EPF. Method
k	2.565	2.649	2.676	2.655
c	7.791	7.83	7.85	7.853

Figure 2. Comparison (Indhumathy and Seshaiah) of 'k' and 'c' for different Methods

Table 5. Error (Indhumathy and Seshaiah) between Measured Wind Speed and Predicted Wind Speed for Different Methods

Various Techniques	Mean wind velocity(m/s)	Error in %
Actual measured value	6.978	0
Equivalent Energy Method	6.974	0.045
ML. Method	6.916	-0.883
Modified ML. Method	6.958	-0.299
EPF. Method	6.976	-0.0158

From the above table, it is seen that there is only slight difference between the measured value and the calculated value by different methods and the error value is lowest in case of E.P.F Method and E.P.F method has simple formulation,it does not require any iterative process.If we have mean wind speed and Wind Power density, We can easily estimate Weibull Parameters also.

4.1 Case Study for W.E.P.F Method:

If we want to Estimate the wind power potential without weibull distribution we can also do so by using the Wind energy pattern factor method. In this method without the help of weibull distribution we can calculate the density of power of wind by simply calculating the EPF for different months or years depending upon the data you have taken either its monthly mean or yearly mean. The Epf is calculated by using the equation (12). After EPF calculation, the wind power density [WPD] is estimated using equation (16). Consider one case study for WEPF method, 1 year data is shown in the below table, then using the equations shown in Energy

Pattern Factor method 'k' and 'c' values are calculated and then we can estimate the value of Power Density as:

$$WPD = \frac{1}{2} \left(W.E.P.F \, \rho \, \bar{v}^3 \right)$$

Table 6. Shows the Variation of E.P.F , 'k' and 'c' with different Wind Speeds (Chandel et al.).

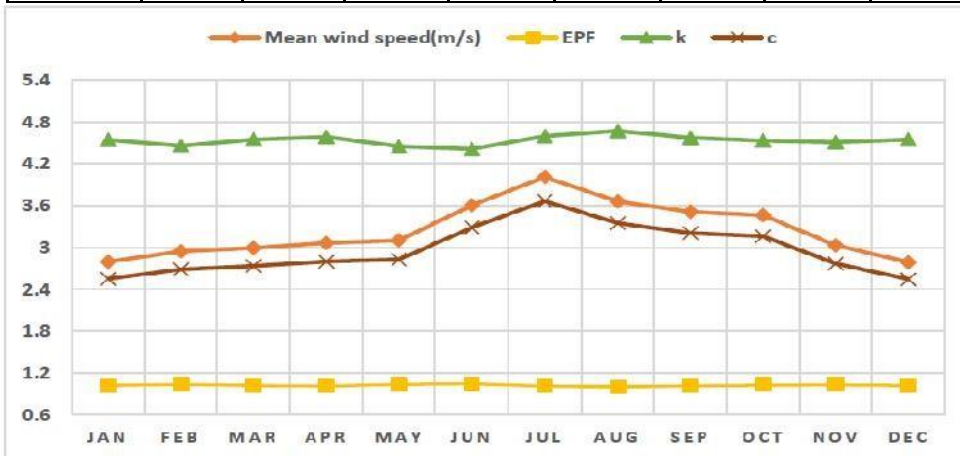| Months	Jan	Feb	Mar	Apr	May	June	Jul	Aug	Sep	Oct	Nov	Dec
Mean wind speed (m/s)	2.795	2.945	2.995	3.063	3.103	3.603	4.01	3.66	3.511	3.463	3.031	2.788
EPF	1.024	1.039	1.023	1.017	1.04	1.047	1.015	1.003	1.019	1.026	1.03	1.023
k	4.543	4.457	4.548	4.584	4.451	4.413	4.596	4.670	4.572	4.531	4.508	4.548
c	2.552	2.686	2.734	2.798	2.830	3.284	3.663	3.347	3.207	3.161	2.766	2.545

Figure 3. Variation of Wind speed , EPF, k and c for different months (Chandel et al.)

4. Conclusions

For the Wind Power Potential estimation analysis, five approaches are described with practical case scenarios. A sample wind speed data set is used to demonstrate the application of each method, and the accuracy is checked for each method. So, it depends on the available data and accuracy check which method we will use. If time series data is available we use the maximum likelihood method, for data in frequency. distribution format we use modified maximum likelihood method, Moment method is generally used to calculate the initial guess iteration value for Maximum and Modified maximum methods and to estimate the Wind Power density without the calculation of Weibull Parameters Wind Energy Pattern Factor is better and accurate as it does not include any iterative process and it has simple formulation.

Nomenclature

W.E.P.F Wind energy Pattern factor.

k Shape parameter

c   Scale parameter

f(v)   Weibull distribution function F(v)   Cumulative density function ρ   Air density in kg/m³

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